CDO Hedging and Risk Management with R

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UseR 2015, Aalborg University, Denmark. June 30 - July 3

Outline

1. Motivation
   - Credit risk instruments in Financial institutions' books.
2. Theoretical framework
   - The insurance contract for exchanging credit risk.
   - Pricing a portfolio of CDO Tranches.
3. Risk quantification of synthetic CDOs
   - Synthetic CDO risk factors.
4. Optimal composition of an hedging tranche
   - Objective function choice.
5. Concluding Remarks

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Credit risk

A borrower can default its loan obligations

- Financial institutions hold many illiquid assets in their books. Credit risk is an intrinsic feature of these assets.
- Credit risk is the risk of loss arising from a borrower who might default its loan obligations.
- Default events are the manifestation of credit risk. They are assumed to happen randomly and at unforeseeable times.
- Default intensity, correlation and contagion effects affect the relevance of credit risk.

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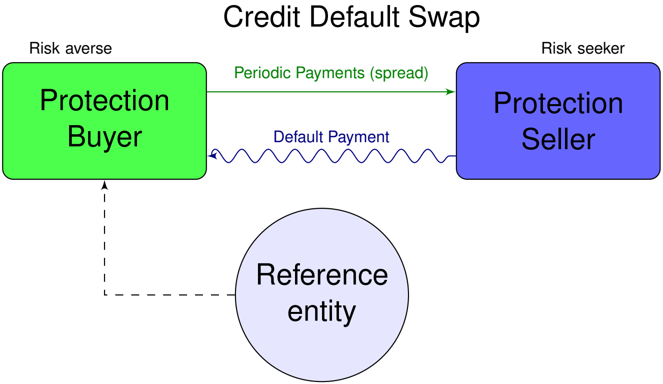
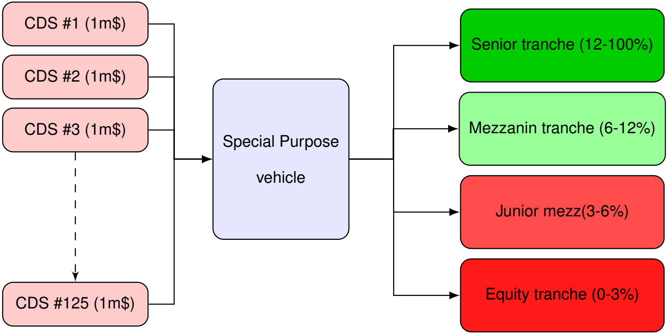
Insurance against a default event.
Single name instrument



Insurance against multiple defaults.

Collateralised Debt Obligation

Synthetic CDO



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A pool of Credit Default Swaps on different names

We consider a portfolio of CDO tranches on an underlying pool of CDS. We make the following assumptions:

- the value of each tranche in the portfolio is computed with a Monte Carlo simulation;
- the total portfolio value is obtained by summing the value of each tranche;
- the spreads and correlation remain constant over the optimization period.

The previous assumptions might be loosened by inserting a dynamics in the spreads and some contagion effects of the obligors default.

Computing the tranche's spread

The loss distribution of each portfolio tranche can be computed employing the following scheme:

Monte Carlo Simulation with Gaussian Copula

procedure CDO LOSS DISTRIBUTION ( )

  for ($\alpha = 1$ to $Nsims$) **do**
    draw $\varepsilon^{\alpha} \sim N(\mathbf{0}, \mathbf{I})$ ▷ Uncorrelated deviates
    compute $\phi^{\alpha} = \mathbf{A} \cdot \varepsilon^{\alpha}$ ▷ **A**: Cholesky factor of disturbance cov matrix
    for (obligor $i = 1$ to n) **do**
      compute $\tau_i = F_i^{-1}(N(\phi))$ ▷ default times for each obligor
      if $(\tau_i \leq T_j)$ **then**
        $\{Loss^{\alpha}_{Pool}(T_j) += (1 - R_i) \cdot Notional_i\}$
    EndFor loop i over obligors
    compute $Loss^{\alpha}_{Cum}(T_j) = \sum_{k=0}^{j} Loss^{\alpha}_{Pool}(T_k)$
  EndFor loop α over replications

  return $(Loss_{Cum})$

Computing the Total portfolio value

Considering the protection buyer standpoint we have:

$$V^{\gamma}(t) = -s^{\gamma} \cdot V^{\gamma}_{Fee}(t) + V^{\gamma}_{Cont}(t) \qquad (1)$$

the spread s^{γ} is computed at contract inception while $V_{Fee}(t)$ and $V_{Cont}(t)$ depend on the tranche losses.
Each of the tranche position can be long (protection buyer) or short (protection seller). For the total portfolio we have:

$$\Pi(\{s_i\}, \rho, J) = \sum_{\gamma=1}^{n_{tranches}} \phi^{\gamma} \cdot V^{\gamma}(\{s_i\}, \rho, J) \qquad (2)$$

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CDO Risk factors

These Credit derivatives instruments are essentially affected by two families of risk factors:

- Market risk factors;
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Market risk factors

The CDO portfolio is exposed to all the risks driving changes in the market value of each tranche:

1. movements in the interest and exchange rates;
2. movements in the credit spread of obligors;
3. movements in the correlations among the obligors;

The last two factors are far more important than the first one.

Credit risk factors

Credit risk factors refer to the event of a default of an obligor in the underlying pool. For a CDO tranche the credit risk depends on:

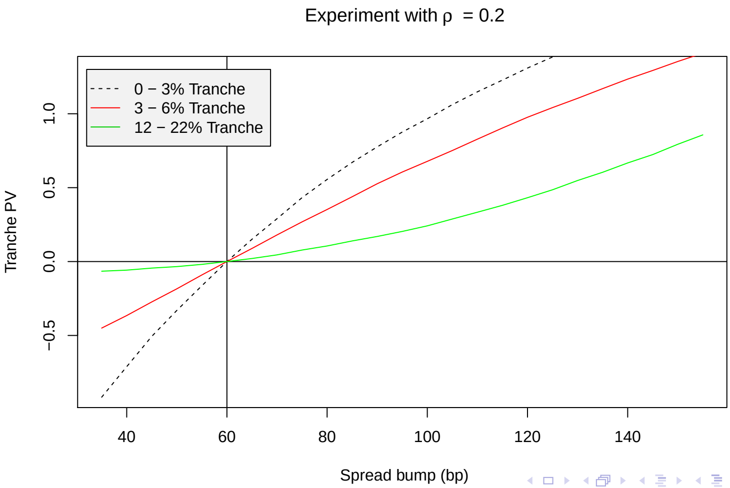
1. tranche attachment point or degree of subordination;
2. tranche thickness;
3. degree of contagion effects in the defaults among obligors;

Credit spread sensitivities.

In the code snippet we show the computation of the PV for three different tranches as function of bumps applied to the spread of the underlying CDS.

```
for (j in ssp) {
  ib        <- j + (-ssp[1] + 1)
  spreade   <- spreadbas  + (j-1)*step
  lambda    <- (spreade )/ fden
  cequit    <- TranPricing(nobl, delta, lambda, rho, Notio[1], c_0,
                            attp, dequi, rfree, Nsim,fleq)
  Vequ_base[ib] <- Vtranche(cequit,spbase,fleq)
  cmezz          <- TranPricing(nobl, delta, lambda, rho, Notio[1], c_0,
                                 amez, dmez, rfree, Nsim,fleq)
   Vmez_base[ib] <- Vtranche(cmezz,spbmez,fleq)
   ..... }
```

Tranche Present Value sensitivity with the spread



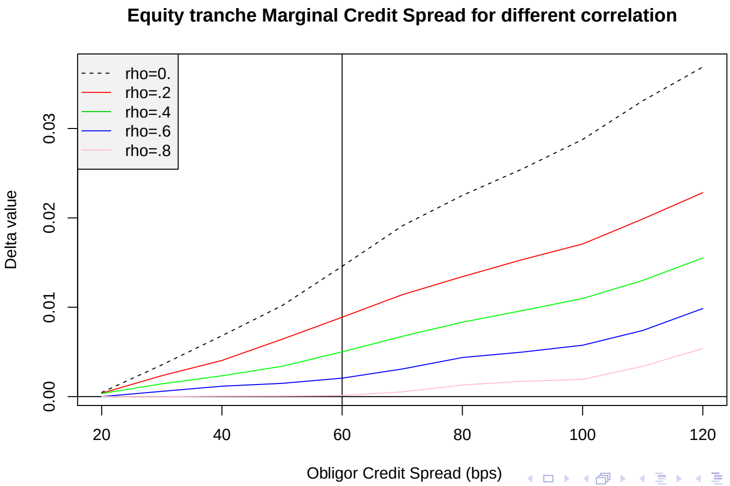
Credit spread sensitivities.

An important credit spread sensitivity figure is the Δ value of the tranche which is defined as

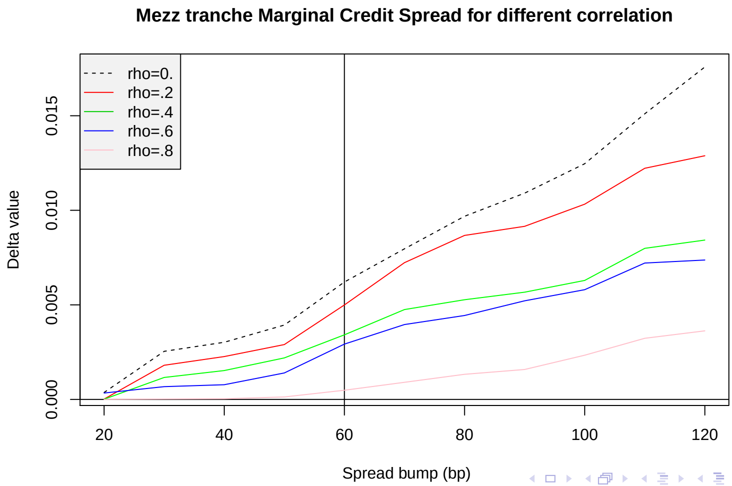
$$\Delta_i^{\gamma} = \frac{\Delta V^{\gamma}(\{s_i\})}{\Delta s_i} \tag{3}$$

where: $\Delta V^{\gamma}(\{s_i\})$ is the change in tranche γ Present Value for a Δs_i bump in the obligor i spread.

Credit spread sensitivities.



Credit spread sensitivities.



Outline

Hedging the portfolio spread sensitivity.

The goal of the optimization exercise is to figure out the composition of a new CDO tranche with the wish to immunize the portfolio P/L against adverse market movements:

$$\mathbb{E}(\hat{J}) = \min_{\{\hat{J}\}} \left(\sum_{\forall \beta} \Pi(\{s_i\}, \beta, \hat{J}) \right)$$

$$s_i = s_i + \beta \cdot s_i$$

where: J is the pool-obligor connectivity matrix, and s_i is the spread on obligor i. Our goal is to minimize the spread sensitivity over a range of spread bumps.

Hedging the portfolio spread sensitivity.

When the number of obligors exceeds 6 or 7 the optimization cannot be tackled with standard derivative based methods. Here we have tried to employ the most popular stochastic heuristic method based on:

- Differential Evolution (**DEoptim**);
- Genetic Algorithms (**ga**);
- Simulation Annealing (**GenSA**);

Hedging the portfolio spread sensitivity.

```
while ( bet < 3) {
     while(it <= ntranc) {
        attp    <- trprop[it,1];    detp    <- trprop[it,2]
        tposit  <- trprop[it,3]
        spcur   <- sp[,it]*(1+bet)
        lambda  <- spcur/(1-delta)*1e-4
        nobl    <- length(lambda)
        if (nobl > 0) {
          tvalue <- TranPricing(nobl, delta, lambda, rho,
          Notio[1], c_0, attp, detp, rfree, Nsim,tposit)
          spbatr  <- spbase[it]
          PVpor <- PVpor + Vtranche(tvalue,spbatr, tposit)  }
          it  <- it + 1     }
        spcur   <- spx * (1 + bet)
        spcur   <- x1*spcur
        lambda  <- spcur /(1-delta)* 1e-4
        attp    <- 0.
        detp    <- 1.   # here I take the whole index [0 - 1]
        nobl    <- length(lambda)
        if (nobl >0 ) {
          finval  <- TranPricing(nobl, delta, lambda, rho,
          Notio[1], c_0, attp, detp, rfree, Nsim,flag)
          PVpor  <- PVpor + Vtranche(finval, spbase, flag) }
        bet <- bet + .25 }
```

Hedging the portfolio spread sensitivity.

Calling the Genetic Algorithm with binary variables.

```
spvar  <- spread
dimension <- length(spvar)
fn.call <- 0
tol <- 1e-3
fitness <-  function(x1,spx,flag,sp,spbase,trprop, rho,ntranc) - myobj(x1,spx,flag,spm,spbase,
sink(file="CDOexa2GA.out",type=c("output"),split=T)

# now we run GA optimization
 GAby <- ga(type = "binary", fitness = fitness, spx,flag, spm, spbase, trprop, rho, nTranc,
     nBits=length(x1),  popSize = 100, pmutation = 0.2, maxiter = 50, run = 20,seed=712343)
summary(GAby)
print(GAby@solution)
```

Hedging the portfolio spread sensitivity.

For these optimizations we have only preliminary results:

- Heuristic optimization algorithms require a careful tuning;
- different seeds should be tested;
- some speed-up technique such as parallelization should be implemented.

Concluding Remarks

- We have written some *R* functions for evaluating portfolio P/L composed of **CDO** tranches;
- we have considered risk management issues in **CDO** portfolios;
- we have written some *R* functions for computing P/L sensitivities to correlation and spread variations;
- we have made a first attempt in employing evolutionary algorithm for computing the tranche composition minimizing the spread sensitivity of a **CDO** portfolio.

For Further Reading

A. De Servigny and N. Jobst.
The Handbook of Structured Finance.
McGraw-Hill, 2007.

G. Löffer and P.N. Posch.
Credit risk modeling using Excel and VBA.
ed. Wiley, 2010.

C.C. Mounfield.
Synthetic CDOs Modelling, Valuation and Risk Management.
Cambridge University Press, 2009.

Our deeper understanding of Credit derivatives

Thank you for your attention.

Tak for din opmærksomhed.

Any questions?